THE AMERICAN STATISTICIAN
2023, VOL. 77, NO. 3, 343–344: Letter to the Editor
https://doi.org/10.1080/00031305.2023.2205455

Response to Comment by Schilling

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ARTICLE HISTORY
Received 10 April 2023
Accepted 17 April 2023

We appreciate the recent paper of Schilling and Stanley (2022, hereafter SS) on confidence intervals for the hypergeometric being brought to our attention, which we were not aware of while preparing our paper (Bartroff, Lorden, and Wang 2022, hereafter BLW) on that subject. Although there are commonalities between the two approaches, there are some important distinctions that we highlight here. Following those papers' notations, below we denote the confidence intervals for the hypergeometric success parameter based on sample size n and population size N by LCO for SS, and $\mathcal{C}^*$ for BLW. In the numerical examples below, LCO (*github.com/mfschilling/HGCIs*) and $\mathcal{C}^*$ (*github.com/bartroff792/hyper*) were computed using the respective authors' publicly available R code, running on the same computer.

Computational time. LCO and $\mathcal{C}^*$ differ drastically in the amount of time required to compute them. Figure 1 shows the computational time of LCO and $\mathcal{C}^*$ for $\alpha = 0.05$, $N = 200, 400, \ldots, 1000$, and $n = N/2$. For example, for $N = 1000$ the computational time of LCO exceeds 100 min whereas $\mathcal{C}^*$ requires roughly 1/10th of a second (0.002 min). In further numerical comparisons not included here, we found this relationship to be common for moderate to large values of the sample and population sizes, n and N. This may be due to the algorithm for computing LCO which calls for searching *among all acceptance functions of minimal span* (SS, p. 37).

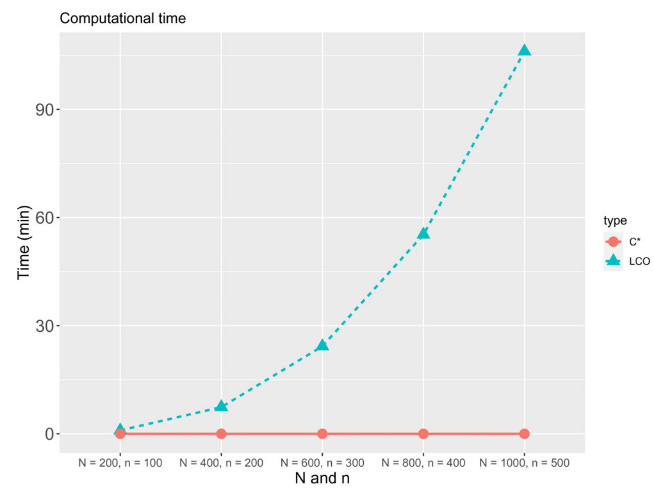

Figure 1. The computational time of the confidence intervals $\mathcal{C}^*$ and LCO for $\alpha = .05$, $N = 200, 400, \ldots, 1000$, and $n = N/2$.

Provable optimality. SS contains two proofs, one in the Appendix of a basic result about the hypergeometric parameters, and one in the main text of the paper's only theorem (SS, p. 33) which is a general result that size-optimal hypergeometric acceptance sets are inverted to yield size-optimal confidence "intervals." However, not all inverted acceptance sets will yield proper intervals, and in practice one often ends up with non-interval confidence *sets*, for example, intervals with "gaps." This occurs when the endpoint sequences of the acceptance intervals being inverted are non-monotonic, or themselves have gaps. SS address this by modifying their proposal in this situation to mimic a method of Schilling and Doi (2014) developed for the Binomial distribution. SS (pp. 36–37) write, *Where there is a need to resolve a gap, in which case the minimal span acceptance function that causes the gap is replaced with the one having the next highest coverage.* This modification is nontrivial in that it, in general, changes both the length and coverage probability of the LCO acceptance and confidence intervals. Since their optimality argument relies on choosing the acceptance functions with the *highest* coverage, does size optimality of LCO still hold when instead choosing the "next highest" coverage, and utilizing a technique for the Binomial that has not been verified for the hypergeometric? These questions are not addressed mathematically in SS, and similar questions remain about their proposed confidence intervals for the population size N. On the other hand, BLW develops a complete optimality theory for $\mathcal{C}^*$, which is substantial (requiring 21 theorems and lemmas in the paper and its supplement) and includes sufficient conditions for when gaps in the optimal confidence sets make $\mathcal{C}^*$ sub-optimal. This is rare and, even when it happens, only causes $\mathcal{C}^*$ to be "too big" by at most a single point.

Symmetry. In hypergeometric inference, any procedure being used should give the same result whether the binary property being counted is considered "success" or "failure," such distinctions being arbitrary. BLW call this property "symmetry" while SS call it "equivariance" and write, *Equivariance is*

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Table 1. The proportion ("% Asymmetric," to 1 decimal place) of LCO upper and lower endpoint pairs that are asymmetric for $\alpha = 0.05$, $N = 200, 400, \ldots, 1000$, and $n = N/2$.

N	200	400	600	800	1000
% Asymmetric	80.2%	76.1%	73.1%	77.1%	73.5%

appropriate when estimating the success parameter of the hypergeometric distribution (SS, p. 35). The $\mathcal{C}^*$ intervals are symmetrical by construction and their optimality theory in BLW takes this into account, which is necessarily more complex because of it. When considering optimality, achieving symmetry is nontrivial and is not just a matter of, say, replacing an asymmetric interval by the reflection of its counterpart since this could change both the interval's length and coverage probability. On the other hand, the LCO intervals are asymmetric by design, and asymmetries occur frequently. In the same setting as Figure 1, Table 1 contains the proportion of LCO's upper and lower endpoint pairs that are asymmetric.

Because of these distinctions between the SS and BLW methods, we hope that readers will regard the two papers as distinct though partially overlapping contributions to the statistics literature, and will consider the optimality results proved in BLW as supporting many of the methodological recommendations in both papers.

Disclosure Statement

The authors report there are no competing interests to declare.

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